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NONSTATIONARY FLOWS OF AN INCOMPRESSIBLE VISCOUS FLUID WITH MEMORY IN CYLINDRICAL TUBES

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UDC 533.6.011

In aerohydromechanical problems, the motion of a viscous thermally conducting gas traditionally is studied with the use of the Navier–Stokes equations, which are the result of the phenomenological closure of the conservation laws on the basis of linear transfer relations connecting the flow of momentum and energy with the spatial gradients of velocity and temperature – that is the transfer laws of Navier–Stokes and Fourier. In the case of slow quasistationary processes, these laws are derived from the kinetic Boltzmann equation with the use of the Chapman–Enskog method [1]. However, it has been shown [2, 3] that in the case of rapid nonstationary motions of a viscous thermally conducting gas, the expressions for the momentum and energy flows should include not only terms with spatial gradients of the velocity and temperature, but also time derivatives (accelerations) of these variables, which characterize the effects of temporal memory. The generalized hdyrodynamic equations [2, 3], which are called hydrodynamic equations for rapid processes, have been used to investigate the distribution of small perturbations, the structure of shock waves, diffusion, etc., and have been used to obtain a series of important results.

In this article, these hydrodynamic equations of rapid processes are used to study the nonstationary motions of a viscous incompressible fluid in circular cylindrical tubes. Exact solutions are found and analyzed for 1) the pulsating motion of the fluid due to a harmonically varying pressure gradient and 2) an instantaneously induced motion of an initially quiescent fluid.

1. For continuous media, the most general form of the laws of conservation of mass, momentum, and energy are written as

$$\frac{\partial \rho}{\partial t}+\frac{\partial \rho u_k}{\partial x_k}=0,\quad \rho\frac{\partial u_i}{\partial t}+\rho u_k\frac{\partial u_i}{\partial x_k}=-\frac{\partial p}{\partial x_i}-\frac{\partial P_{ik}}{\partial x_k},$$
$$\rho\frac{\partial e}{\partial t}+\rho u_k\frac{\partial e}{\partial x_k}=-p\frac{\partial u_k}{\partial x_k}-P_{ik}\frac{\partial u_i}{\partial x_k}-\frac{\partial Q_k}{\partial x_k}, \tag{1.1}$$

where ρ is the density; u_i (i = 1, 2, 3) are the velocity components along the x_i axis of the Cartesian coordinate system (x_1, x_2, x_3); p is the pressure, e is the internal energy; P_{ik} is the momentum flux (stress tensor); and Q_i is the thermal flux (energy flux). In order to obtain a closed system of equations from the conservation laws (1.1), the momentum and energy fluxes must be expressed in terms of parameters of the hydrodynamic state ρ, u_i, and e.

Zhukovskii. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 5, pp. 52-57, September-October, 1991. Original article submitted May 30, 1990.

Within the framework of Newtonian mechanics of continuous media, the phenomenological linear transfer equations of Navier–Stokes and Fourier are used: $P_{ij} = -\mu D_{ij}$, $Q_i = -\lambda \partial T/\partial x_i$. Here μ and λ are the coefficients of viscosity and thermal conductivity; and $D_{ij} = \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3}\frac{\partial u_k}{\partial x_k}\delta_{ij}$ is the strain rate tensor.

It has been shown [3, 4] that these laws correctly describe slow quasistationary motions and that for describing rapidly time-varying processes the linear algebraic relations of the fluxes to the gradients should be replaced by differential relations

$$\tau_p\left(\frac{\partial P_{ij}}{\partial t} + u_k\frac{\partial P_{ij}}{\partial x_k}\right) = -P_{ij} - \mu D_{ij},$$

$$\tau_q\left(\frac{\partial Q_i}{\partial t} + u_k\frac{\partial Q_i}{\partial x_k}\right) = -Q_i - \lambda\frac{\partial T}{\partial x_i},$$

where τ_p and τ_q are relaxation times. In the case of a monatomic perfect gas, $\tau_p = \mu/p$ and $\tau_q = \lambda/c_p p$, where c_p is the heat capacity at constant pressure.

2. In order to obtain some idea on the changes which make use of the hydrodynamics of rapid processes as opposed to the Navier–Stokes hydromechanics, two problems of nonstationary hydrodynamics are examined in this section and the following sections.

First we study the nonstationary motion of an incompressible viscous fluid through a circular cylindrical tube (for a classical formulation and solution, see [5]). For flows of an incompressible fluid, the total system of equations for the hydromechanics of rapid processes takes the form

$$\frac{\partial u_k}{\partial x_k} = 0, \quad \rho\left(\frac{\partial u_i}{\partial t} + u_k\frac{\partial u_i}{\partial x_k}\right) = -\frac{\partial p}{\partial x_i} - \frac{\partial P_{ik}}{\partial x_k},$$
$$\tau\left(\frac{\partial P_{ij}}{\partial t} + u_k\frac{\partial P_{ij}}{\partial x_k}\right) = -\mu D_{ij} - P_{ij}.$$

The parameter τ and the viscosity μ are considered constant. We align the $x_3 = z$ axis with the axis of the tube and the $x_1 = x$ and $x_2 = y$ axes in the plane of the transverse cross section. If we set $u_1 = u_2 = 0$, and $u_3 = w$, the continuity equations give $\partial w/\partial z = 0$, and $w = w(x, y)$. The remaining pair of equations gives

$$\rho\partial w/\partial t = -\partial p/\partial z - \partial P_{zx}/\partial x - \partial P_{zy}/\partial y,$$
$$(1 + \tau\partial/\partial t)P_{ij} = -\mu D_{ij} \quad ((ij) = (zx) \text{ or } (zy)),$$

from which it follows that

$$\left(1 + \tau\frac{\partial}{\partial t}\right)\left(\frac{\partial w}{\partial t} + \frac{1}{\rho}\frac{\partial p}{\partial z}\right) = \nu\Delta_2 w. \tag{2.1}$$

Here Δ_2 is the Laplacian operator in the plane transverse cross section, which in polar coordinates r, ϕ gives

$$\Delta_2 w = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right),$$

because w does not depend on ϕ. Equation (2.1) is the basic equation for nonstationary motion of an incompressible viscous fluid with memory through a cylindrical tube. It makes it possible to find the distribution of the longitudinal velocity through a tube cross section for a given pressure gradient $\partial p/\partial z$ as a function of time t.

Now we examine a pulsating motion, which corresponds to a harmonic law for the change in the pressure gradient in the tube: $-\partial p/\partial z = \rho A \cos \omega t$, where $A = \text{const}$. Then (2.1) takes the form

$$\left(1 + \tau\frac{\partial}{\partial t}\right)\left(\frac{\partial w}{\partial t} - A\cos\omega t\right) = \frac{\nu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right).$$

We will investigate the steady-state regime. The initial conditions now lose their meaning, and we need only satisfy the boundary condition of attachment at the wall: w = 0 for r = a (a is the radius of the tube). We assume

$$w(r, t) = u(r, t) + (A/\omega) \sin \omega t. \tag{2.2}$$

Then the problem reduces to the solution of the equation

$$\left(1 + \tau \frac{\partial}{\partial t}\right) \frac{\partial u}{\partial t} = \frac{\nu}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u}{\partial r}\right) \tag{2.3}$$

with the boundary condition

$$u(a, t) = -(A/\omega) \sin \omega t. \tag{2.4}$$

We apply the method of separation of variables in the form

$$u(r, t) = \mathrm{Re}[CR(r) \exp(i\omega t)], \tag{2.5}$$

where C is a constant. From (2.3) and (2.5) it follows that

$$r^{-1}(rR')' + i\alpha R = 0, \ \alpha = (\omega/\nu)(1 - i\omega\tau). \tag{2.6}$$

The solution of (2.6) which is finite at r = 0 (the axis of the tube) has the form

$$R(r) = J_0(r\sqrt{i\alpha}), \tag{2.7}$$

where $J_0(x)$ is the Bessel function for the first kind of zero order of the complex argument x [6], which is defined, for example, by the series

$$J_0(x) = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k!!)^2} x^{2k}.$$

In accordance with (2.6) and (2.7), the argument x is represented as $x = r\sqrt{i\alpha} = r\left(\frac{\omega}{\nu}\right)^{1/2}(1 + \omega^2\tau^2)^{1/4} \exp\left[i\left(\frac{\pi}{4} - \frac{\beta}{2}\right)\right]$ ($\beta = \mathrm{arctg}\ \omega\tau$). Consequently

$$x^{2k} = \left(r^2 \frac{\omega}{\nu} \sqrt{1 + \omega^2\tau^2}\right)^k \exp\left[ik\left(\frac{\pi}{2} - \beta\right)\right].$$

Thus, (2.7) has the form $R(r) = Z_0(r) - iZ_1(r)$, where the real functions Z_0 and Z_1 are given by the series

$$Z_0(r) = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k!!)^2} \left(r^2 \frac{\omega}{\nu} \sqrt{1 + \omega^2\tau^2}\right)^k \cos\left(\frac{k\pi}{2} - \beta k\right),$$

and

$$Z_1(r) = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k!!)^2} \left(r^2 \frac{\omega}{\nu} \sqrt{1 + \omega^2\tau^2}\right)^k \sin\left(\frac{k\pi}{2} - \beta k\right).$$

We note that for $\tau = 0$

$$[Z_0(r)]_{\tau=0} = \sum_{n=0}^{\infty} \frac{(-1)^n}{(4n!!)^2} \left(r^2 \frac{\omega}{\nu}\right)^{2n} = \mathrm{ber}\left(r\sqrt{\frac{\omega}{\nu}}\right),$$

and

$$[Z_1(r)]_{\tau=0} = \sum_{n=0}^{\infty} \frac{(-1)^n}{[(4n+2)!!]^2} \left(r^2 \frac{\omega}{\nu}\right)^{2n+1} = \mathrm{bei}\left(r\sqrt{\frac{\omega}{\nu}}\right),$$

where ber(x) and bei(x) are the Kelvin functions [6].

Now we write the solution of Eq. (2.3) in the form of the real part of the expression (2.5) with a complex constant $C = C_r + iC_i$:

$$u(r,\ t) = (C_r Z_0 + C_i Z_1) \cos \omega t + (C_i Z_0 - C_r Z_1) \sin \omega t.$$

From the boundary condition (2.4) it follows that

$$C_r = \frac{A}{\omega} \frac{Z_{1a}}{Z_{0a}^2 + Z_{1a}^2}, \quad C_i = -\frac{A}{\omega} \frac{Z_{0a}}{Z_{0a}^2 + Z_{1a}^2}$$
$$(Z_{0a} = Z_0(a),\ Z_{1a} = Z_1(a)).$$

Substituting these rsults in (2.2) yields

$$w(r,\ t) = \frac{A}{\omega} \left\{ \left[1 - \frac{Z_{0a}}{Z_{0a}^2 + Z_{1a}^2} Z_0(r) - \frac{Z_{1a}}{Z_{0a}^2 + Z_{1a}^2} Z_1(r) \right] \sin \omega t + \right.$$
$$\left. + \left[\frac{Z_{1a}}{Z_{0a}^2 + Z_{1a}^2} Z_0(r) - \frac{Z_{0a}}{Z_{0a}^2 + Z_{1a}^2} Z_1(r) \right] \cos \omega t \right\}. \tag{2.8}$$

We note that for $\tau = 0$, Eq. (2.8) transforms into the corresponding solution of the Navier–Stokes equations [5]. In this case the hydrodynamics of rapid processes is identical to the hydrodynamics of a Newtonian fluid with a Maxwellian rheological viscosity law.

3. We will examine the problem of setting into motion an intially quiescent incompressible viscous fluid in a cylindrical tube by instantaneously applying a given pressure drop. The Navier–Stokes solution of this problem can be found in [7], for example.

Within the framework of the hydrodynamics of rapid processes, the velocity $w(r,\ t)$ along the z-axis first of all satisfies (2.1), in which $\partial p/\partial z = -\Delta p/\ell$ (Δp is the pressure drop over a length ℓ of the tube), however, the derivation of a unique solution requires one more initial condition in addition to the classical conditions $w = 0$ for $t = 0$ and $w = 0$ for $r = a$; for example the acceleration $\partial w/\partial t$ for $t = 0$. This question will be examined in more detail later, after we solve the problem for arbitrary initial conditions.

We write the solution to the problem

$$\left(1 + \tau \frac{\partial}{\partial t}\right)\left(\frac{\partial w}{\partial t} - \frac{\Delta p}{\rho l}\right) = \frac{\nu}{r} \frac{\partial}{\partial r}\left(\frac{\partial w}{\partial r}\right)$$

in the form

$$w(r,\ t) = u(r,\ t) + \frac{a^2 \Delta p}{4 \mu l}\left(1 - \frac{r^2}{a^2}\right). \tag{3.1}$$

Then the function u satisfies the homogeneous equation

$$\left(1 + \tau \frac{\partial}{\partial t}\right)\frac{\partial u}{\partial t} = \frac{\nu}{r} \frac{\partial}{\partial r}\left(r \frac{\partial u}{\partial r}\right) \tag{3.2}$$

with the conditions

$$u(a,\ t) = 0, \quad u(r,\ 0) = -\frac{a^2 \Delta p}{4 \mu l}\left(1 - \frac{r^2}{a^2}\right).$$

We use the method of separation of variables and assume $u = T(t)R(r)$. From (3.2) we have

$$\frac{\tau \ddot{T}(t) + \dot{T}(t)}{T} = \frac{\nu}{rR}(rR')' = -\alpha, \tag{3.3}$$

where the dots and primes denote derivatives with respect to t and r, respectively, and α is a positive constant. From (3.3) we find $R(r) = J_0(r\sqrt{\alpha/\nu})$. Using the boundary condition $R(a) = 0$, we find that the parameter α can take on discrete values $\alpha_k = \nu\lambda_k^2/a^2$, $k = 1, 2, 3, \ldots$, where λ_k is a zero of the Bessel function $J_0(\lambda_k) = 0$. Now we follow the standard procedure $\tau\ddot{T}_k + \dot{T}_k + \alpha_k T_k = 0$ and $T_k = A_k \exp(s_{1k}t) + B_k \exp(s_{2k}t)$. Here $s_{1k} = (-1 + \sqrt{1 - 4\alpha_k\tau})/2\tau$, and $s_{2k} = (-1 - \sqrt{1 - 4\alpha_k\tau})/2\tau$.

Thus,

$$u(r,t)=\sum_{k=1}^{\infty}[A_k\exp(s_{1k}t)+B_k\exp(s_{2k}t)]\,J_0\left(\lambda_k\frac{r}{a}\right). \tag{3.4}$$

According to the initial condition

$$u(r,0)=-\frac{a^2\Delta p}{4\mu l}\left(1-\frac{r^2}{a^2}\right)=\sum_{k=1}^{\infty}(A_k+B_k)\,J_0\left(\lambda_k\frac{r}{a}\right). \tag{3.5}$$

In order to determine the coefficient $A_k + B_k$, we use the results of the theory of Fourier–Bessel series, according to which an arbitrary function f(x) of a real variable x can be represented in the form of a series

$$f(x)=\sum_{k=1}^{\infty}a_kJ_0(\lambda_kx), \tag{3.6}$$

where

$$a_k=\frac{2}{[J_1(\lambda_k)]^2}\int_0^1 xf(x)\,J_0(\lambda_kx)\,dx. \tag{3.7}$$

Equation (3.7) is a result of the orthogonality of Bessel functions:

$$\int_0^1 xJ_\nu(\lambda_kx)\,J_\nu(\lambda_mx)\,dx=\frac{[J_{\nu+1}(\lambda_k)]^2}{2}\,\delta_{km};$$

where λ_k is a root of the equation $J_\nu(\lambda_k) = 0$.

We multiply (3.5) by $xJ_0(\lambda_kx)$, where $x = r/a$, and integrate over x from 0 to 1. In accordance with (3.7), we obtain

$$A_k+B_k=\frac{-a^2\Delta p}{2\mu l\,[J_1(\lambda_k)]^2}\int_0^1(x-x^3)\,J_0(\lambda_kx)\,dx. \tag{3.8}$$

In calculating the integral in (3.8) we use the series representation of $J_p(x)$:

$$J_p(x)=\sum_{s=0}^{\infty}\frac{(-1)^s}{s!\,(s+p)!}\left(\frac{x}{2}\right)^{2s+p},$$

which we substitute into (3.8) and find

$$\int_0^1(x+x^3)\,J_0(\lambda_kx)\,dx=\frac{2}{\lambda_k^2}\sum_{s=0}^{\infty}\frac{(-1)^s\lambda_k^{2s+2}}{s!\,(s+2)!\,2^{2s+2}}=\frac{2J_2(\lambda_k)}{\lambda_k^2}.$$

We substitute this result into (3.8), use the formula $J_2(x) = (2/x)J_1(x) - J_0(x)$ with $x = \lambda_k$ and the condition $J_0(\lambda_k) = 0$, and have

$$A_k+B_k=-\frac{2a^2\Delta p}{\mu l\lambda_k^3J_1(\lambda_k)}. \tag{3.9}$$

Finding an additional relationship between the constants A_k and B_k requires formulating one more initial condition, for example the value of the acceleration $\partial w/\partial t$ at $t = 0$. In the case of Navier–Stokes hydrodynamics, this quantity is obtained as a result of the solution

$$\partial w/\partial t|_{t=0}=\Delta p/\rho l, \tag{3.10}$$

where now it can be specified arbitrarily. In order to compare with the Navier–Stokes solution, we formulate the missing condition in the form of (3.10). Then

$$\left.\frac{\partial u}{\partial t}\right|_{t=0}=\frac{\Delta p}{\rho l}=\sum_{k=1}^{\infty}(A_ks_{1k}+B_ks_{2k})\,J_0(\lambda_kx). \tag{3.11}$$

We use the known formula $\int xJ_\nu(x)\,dx = xJ_{\nu+1}(x)$ with $\nu = 0$, and find from (3.6) and (3.7) that

$$1 = \sum_{k=1}^{\infty} \frac{2J_0(\lambda_k x)}{\lambda_k J_1(\lambda_k)}. \tag{3.12}$$

According to (3.11) and (3.12)

$$A_k s_{1k} + B_k s_{2k} = \frac{2\Delta p}{\rho l \lambda_k J_1(\lambda_k)}. \tag{3.13}$$

From Eqs. (3.9) and (3.13) we find the constants A_k and B_k:

$$A_k = \frac{2\Delta p\tau (1 + s_{2k}/\alpha_k)}{\rho l \lambda_k J_1(\lambda_k)\sqrt{1-4\alpha_k\tau}}, \quad B_k = \frac{-2\Delta p\tau (1 + s_{1k}/\alpha_k)}{\rho l \lambda_k J_1(\lambda_k)\sqrt{1-4\alpha_k\tau}}.$$

We note that for $4\alpha_k\tau > 1$, the quantities s_{1k} and s_{2k} are complex; nonetheless the expression (3.4) remains real. This can be proved easily, noting that $s_{1k} = \overline{s_{2k}}$. The corresponding terms of the series contain oscillating components with attenuation in the form

$$\exp(-t/2\tau)(a_k \cos\omega_k t + b_k \sin\omega_k t), \quad \omega_k = \frac{\sqrt{|1-4\alpha_k\tau|}}{2\tau}.$$

Now we examine the case of small τ's. We expand the results in a Taylor series and obtain

$$s_{1k} = -\alpha_k - \alpha_k^2\tau + O(\tau^2), \quad s_{2k} = -\tau^{-1} + \alpha_k + O(\tau),$$
$$A_k = \frac{-2\Delta p a^2}{\mu l \lambda_k^3 J_1(\lambda_k)}[1 + O(\tau^2)], \quad B_k = \frac{2\Delta p \alpha_k \tau^2}{\rho l \lambda_k^3 J_1(\lambda_k)}[1 + O(\tau)].$$

Substituting these estimates into (3.1) and (3.4), we find to an accuracy $O(\tau^2)$:

$$w(r,t) = \frac{a^2\Delta p}{4\mu l}\left\{1 - \frac{r^2}{a^2} - 8\sum_{k=1}^{\infty} \frac{J_0\left(\lambda_k \frac{r}{a}\right)}{\lambda_k^3 J_1(\lambda_k)} \exp[-\alpha_k t(1+\alpha_k\tau)]\right\}.$$

We set $\tau = 0$ and find the Navier–Stokes solution to the problem [7]:

$$w(r,t) = \frac{a^2\Delta p}{4\mu l}\left[1 - \frac{r^2}{a^2} - 8\sum_{k=1}^{\infty} \frac{J_0\left(\lambda_k \frac{r}{a}\right)}{\lambda_k^3 J_1(\lambda_k)} \exp(-\nu\lambda_k^2 t/a^2)\right].$$

Thus, for small values of τ, the difference between our solution and the Navier–Stokes solution is basically contained in the exponent (at least in terms of order less than τ^2). Expanding the exponent to terms of order τ would give a secular term of the type τt.

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EFFECT OF INSTABILITY ON BOUNDARY LAYER DETACHMENT

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UDC 533.6.011

When fluid or gas flows around a body, a thin boundary layers forms near its surface. The behavior of this boundary layer is determined by hydrodynamic resistance. If the boundary layer is detached from the surface, the resistance increases sharply [1, 2]. In order to reduce this resistance, the detachment must be stretched out; that is the boundary layer line detachment must be shifted as far as possible to the aft critical point, so that the region of stagnant flow (the wake) behind the body is narrowed. In this regard, investigations of nonstationary fluid around a body are of current interest. The acceleration of a cylindrical body into a quiescent fluid has been examined [2]. Undetached flow around the body was observed immediately after the acceleration started. Then, after the cylinder traveled a distance s = 0.351 R (where R is the cylinder radius), the flow detached near the aft critical point of the body. The detachment gradually moved forward along the flow and increased the wake behind the body. After a certain time, a pair of vortices appeared behind the body, which grew and continually broke off to form a vortical wake. As measurements show [3], the hydrodynamic resistance coefficient is minimized in the case of undetached flow. This leads to the importance of investigating nonstationary flow around bodies.

Currently, boundary layer dynamics for bodies accelerating into a flow of fluid or gas have not been studied enough. Here it must be kept in mind that acceleration into a quiescent fluid is different than into a flowing one, where, as a rule, the boundary layer detachment already exists, and it is necessary to follow its behavior as the body accelerates.

It has been shown [1, 4] that for a nonstationary boundary layer, the velocity profile inside it is defined by the parameter

$$\lambda = (\delta^2/\nu)(U' + \dot{U}/U),$$

where δ is the boudnary layer thickness; ν is the kinematic viscosity; U is the flow velocity at the boundary with the boundary layer; $U' = \partial U/\partial x$ is the velocity gradient (x is the coordinate along the arc of the meridional cross section of the body); and $\dot{U} = \partial U/\partial t$ is the time derivative of the velocity (acceleration). This equation shows that the growth rate of the boundary layer, its structure, and the position of the detachment line will depend on the magnitude and sign of the relative acceleration $\dot{U}/U$, which in the nonstationary boundary layer plays the same role as U'.

The investigations were conducted in a hydrodynamic tunnel whose working section is a square channel 40 × 40 mm made of transparent material. A device which can accelerate the body into the flow is mounted in the working section (Fig. 1). The device is constructed as follows. The flow body (a cylinder) 1 is fastened to the end of a steel tube 2, which passes through the body to the forward point on the cylinder. The tube carries water with a fluorescent dye to the forward point, in order to make the boundary layer visible. In turn, the tube 2 passes through a directing tube 3, which is rigidly fastened to a support 4, which is fastened from two opposite sides to the channel walls 5. A mounting assembly 6 is rigidly connected to the tube 2. A control thread 7 and the flow cylinder, are pulled back along guide wires 8, which prevent the cylinder form rotating around the axis, and stretch springs 9. At a given moment in time, the thread is broken and the flow body is accelerated by the compressing spring into the flow.

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 5, pp. 57-62, September-October, 1991. Original article submitted May 3, 1990.

0021-8944/91/3205-0703$12.50